LESSON 6.1b

Writing Exponential Models

Today you will:

- Write exponential models to analyze real world situations.
- Practice using English to describe math processes and equations

Core Vocabulary:

- Exponential function, p. 296
- Exponential growth function, p. 296
- Growth factor , p. 296
- Exponential decay function , p. 296
- Decay factor , p. 296
- Asymptote , p. 296

Previous:

- Properties of exponents

Exponential Models (review)

Exponential equations/functions that reflect (model) real-world situations.

A common example is a value you are tracking that increases or decreases by a fixed percentage over a regular period of time (a year).

Exponential Growth Model

$$y = a(1 + r)^t$$

Exponential Decay Model

$$y = a(1 - r)^t$$

…where a is the initial/starting value

r is the rate of change … percent growth or decay

t is the amount of time that has past

$(1 + r)$ is the growth factor

$(1 - r)$ is the decay factor

In 2000, the world population was about 6.09 billion. During the next 13 years, the world population increased by about 1.18% each year.

a. Write an exponential growth model giving the population y (in billions) t years after 2000. Estimate the world population in 2005.

b. Estimate the year when the world population was 7 billion.

SOLUTION

a. The initial amount is $a = 6.09$, and the percent increase is $r = 0.0118$. So, the exponential growth model is

$$y = a(1 + r)^t$$ Write exponential growth model.

$$= 6.09(1 + 0.0118)^t$$ Substitute 6.09 for a and 0.0118 for r.

$$= 6.09(1.0118)^t.$$ Simplify.

Using this model, you can estimate the world population in 2005 ($t = 5$) to be $y = 6.09(1.0118)^5 \approx 6.46$ billion.

X	Y1	
6	6.5341	
7	6.6112	
8	6.6892	
9	6.7681	
10	6.848	
11	6.9288	
12	7.0106	

X=12

b. Use the *table* feature of a graphing calculator to determine that $y \approx 7$ when $t = 12$. So, the world population was about 7 billion in 2012.

Tell whether the function represents exponential growth or decay.
What is the rate of growth or decay?

$y = (1.3)^x$

SOLUTION

$y = (1.3)^x$ — $b = 1.3$ (greater than 1) so this is growth

$y = (1 + .3)^x$ — Rewrite in $y = (1 + r)^x$ form, r is the rate of growth

$r = .3$

▶ The rate of growth is .3 or 30%.

Tell whether the function represents exponential growth or decay.
What is the rate of growth or decay?

$$y = \left(\frac{2}{3}\right)^x$$

SOLUTION

$$y = \left(\frac{2}{3}\right)^x$$

$b = \frac{2}{3}$ (less than 1) so this is decay

Rewrite in $y = (1 - r)^x$ form, r is the rate of decay

$$\frac{2}{3} = (1 - r)$$

$b = (1 - r)$

$$r = 1 - \frac{2}{3}$$

Solve for r

$$r = \frac{1}{3}$$

The rate of decay is $\frac{1}{3}$ or $.\overline{33}$ or 33%.

The amount y (in grams) of the radioactive isotope chromium-51 remaining after t days is $y = a(0.5)^{t/28}$, where a is the initial amount (in grams). What percent of the chromium-51 decays each day?

SOLUTION

$y = a(0.5)^{t/28}$	Write original function.
$= a[(0.5)^{1/28}]^t$	Power of a Power Property
$\approx a(0.9755)^t$	Evaluate power.
$= a(1 - 0.0245)^t$	Rewrite in form $y = a(1 - r)^t$.

The daily decay rate is about 0.0245, or 2.45%.

You deposit $9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded quarterly.

SOLUTION

With interest compounded quarterly (4 times per year), the balance after 3 years is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ Write compound interest formula.

$$= 9000\left(1 + \frac{0.0146}{4}\right)^{4 \cdot 3}$$ $P = 9000$, $r = 0.0146$, $n = 4$, $t = 3$

$$\approx 9402.21.$$ Use a calculator.

▶ The balance at the end of 3 years is $9402.21.

Homework

Pg 300, #23-44